

Constrained Standard Model from Extra Dimension

Yasunori Nomura*

*Department of Physics,
and
Theoretical Physics Group, Lawrence Berkeley National Laboratory,
University of California, Berkeley, CA 94720*

Abstract

A five dimensional supersymmetric model is constructed which reduces to the one Higgs-doublet standard model at low energies. The radiative correction to the Higgs potential is finite and calculable, allowing the Higgs mass prediction of 127 ± 8 GeV. The physical reasons for this finiteness are discussed in detail. The masses of the superparticles and the Kaluza-Klein excitations are also predicted. The lightest superparticle is a top squark of mass 197 ± 20 GeV.

*Research Fellow, Miller Institute for Basic Research in Science.

1 Introduction

The standard model extremely well describes the physics down to a scale of 10^{-18} m. To achieve the electroweak symmetry breaking (EWSB), it employs the Higgs sector which consists of one SU(2) doublet scalar field h with the following Lagrangian,

$$\mathcal{L}_{\text{Higgs}} = -m_h^2|h|^2 - \lambda_h|h|^4, \quad (1)$$

where m_h^2 is assumed to be negative to trigger EWSB. This Higgs sector is the least explored sector of the standard model and has the following unpleasant features:

- The radiative correction δm_h^2 to the Higgs mass-squared parameter is quadratically divergent, $\delta m_h^2 \propto -\Lambda^2$ (Λ is a cutoff scale), and completely dominated by unknown ultraviolet physics. It means that we cannot reliably calculate the crucial quantity m_h^2 for EWSB, and thus the standard model does not provide the theory of EWSB. Also, if the cutoff scale Λ is large as in the conventional grand desert scenario, an extreme fine-tuning between the tree and loop level contributions is required to maintain the Higgs vacuum expectation value (VEV) at the weak scale.
- The Lagrangian Eq. (1) contains two free parameters, m_h^2 and λ_h . Since the observed Fermi constant determines only one linear combination, $\langle h \rangle^2 = |m_h^2/(2\lambda_h)| \simeq (174 \text{ GeV})^2$, the other combination remains unfixed and we cannot predict the physical Higgs boson mass. Although this is not a problem of the theory, it is somewhat unpleasant from the viewpoint of the predictivity of the model.

Supersymmetry (SUSY) elegantly solves the problem of quadratic divergence. If we extend the standard model to be supersymmetric, the radiative correction to the Higgs mass cancels between the loops of bosons and fermions, so that the weak-scale Higgs mass becomes stable quantum mechanically. However, since no degeneracy between bosons and fermions is seen in nature, SUSY must be a broken symmetry. The size of the SUSY-breaking masses should be of the order of the weak scale to keep the Higgs mass at the weak scale.

This leads to the usual framework beyond the standard model, specifically to the minimal SUSY standard model (MSSM) or its extensions. Since the quadratic divergence of δm_h^2 cancels in these theories, we can make the cutoff scale Λ large, say the Planck scale, without introducing enormous fine-tuning. Furthermore, the radiative correction δm_h^2 can be calculated and is negative due to top/stop loops [1]. Therefore, we have a theory of EWSB.

Although the above framework is a promising candidate of the physics beyond the standard model, there are still several features which are not completely satisfactory. They are,

- Since we need soft SUSY-breaking parameters of the order of the weak scale, we have to answer how SUSY is broken. It is known that, to obtain a realistic spectrum, SUSY-breaking sector must be separated from the MSSM sector, and the breaking must be

transmitted to the MSSM sector through some interactions [2]. This makes the structure of the theory somewhat complicated.

- Although the quadratic divergence in δm_h^2 cancels between bosonic and fermionic loops, there is still a residual logarithmic divergence. This means that the contributions to δm_h^2 come from all energies between the weak and the cutoff scales, so that the scale of the EWSB physics is unclear in this type of models.
- In supersymmetric models, quartic couplings of the Higgs fields are generically related to the standard model gauge couplings: $\lambda_h \sim (g^2 + g'^2)/8$ where g and g' are the SU(2) and U(1) gauge couplings, respectively. However, SUSY also requires two Higgs doublets to give both up- and down-type quark masses and to cancel gauge anomalies coming from Higgsinos. This introduces extra free parameters such as the ratio of the VEVs of the two Higgs doublets. Thus, the physical Higgs boson mass is still not predicted (although there is an upperbound coming from the fact that λ_h 's are determined by SUSY).

The question then is whether these conclusions are really unavoidable in *any* SUSY extension of the standard model.

In fact, to reach the above conclusions, we have implicitly assumed that nature is four dimensional up to high energy scales such as the Planck scale. More precisely, there is an energy interval where the physics is described by a four dimensional $\mathcal{N} = 1$ supersymmetric theory. However, we now know that it does not necessarily have to be true. For instance, if there are large extra dimensions in which only gravity propagates, the fundamental scale of nature can be a TeV scale [3]. The observed weakness of the gravity then is explained by a wavefunction suppression of the graviton due to a large volume of the extra dimensions.

One might think that there is no need for SUSY in this case, since the divergence of the Higgs mass is cut off at the TeV scale. However, without SUSY, the divergence is still quadratic, so that we even do not know the sign of m_h^2 in effective field theory. Therefore, unless we embed the theory into a more fundamental theory such as string theory and calculate δm_h^2 , the framework itself does not provide a theory of EWSB. We still need SUSY to control the Higgs mass and have a theory of EWSB at the effective field theory level.

However, lowering the fundamental scale down to a TeV scale opens up a wide variety of possibilities as to how to implement SUSY to the standard model. In particular, we can consider a TeV-sized extra dimension in which the standard-model gauge, quark and lepton multiplets propagate. That is, the theory has a higher dimensional SUSY above the TeV scale. (The TeV-scale extra dimension has been considered in Ref. [4].) Then, we find that the previous conclusions are no longer necessarily correct. Specifically, we can construct a model [5] in which

- We do not need any hidden sector to break SUSY. SUSY is broken by a compactification [6], and the masses for the superparticles are determined by the radius R of the extra dimension [7, 5, 8].

- The radiative correction to the Higgs mass-squared parameter (the Higgs effective potential V_h) is completely finite and calculable in terms of the compactification radius R [8, 5].
- The low-energy effective theory is precisely one Higgs-doublet standard model with the tree-level Higgs potential constrained as

$$V_{h,\text{tree}} = \frac{g^2 + g'^2}{8} |h|^4. \quad (2)$$

Since the tree-level Higgs potential does not contain any free parameter and the radiative correction is calculated in terms of one parameter R , we can determine the compactification radius R by requiring the Higgs VEV to be $\langle h \rangle \simeq 174$ GeV. It also determines the physical Higgs boson, superparticle and Kaluza-Klein (KK) excitation masses. In the next section, we briefly review the structure of five dimensional SUSY and an orbifold compactification. The explicit model of Ref. [5] is explained in section 3. Finally, section 4 is devoted to our conclusions.

2 Five Dimensional Supersymmetry

In this article, we consider supersymmetric models in five dimensions (5D) where all the standard-model fields propagate in the bulk. The SUSY multiplets in 5D are larger than those in four dimensions (4D). The standard model matter and Higgs fields are described by a set of hypermultiplets, and each hypermultiplet contains two complex scalars and two Weyl fermions. For instance, the standard-model quark q is accompanied by a squark \tilde{q} , a conjugate squark \tilde{q}^c and a conjugate quark q^c . Here, conjugated objects have conjugate transformations under the gauge group. The standard model gauge bosons A_μ ($\mu = 0, 1, 2, 3$) are contained in 5D vector supermultiplets $(A_M, \lambda, \lambda^c, \sigma)$ where $M = \mu, 5$.

Upon compactifying the fifth dimension y on the circle S^1 , each 5D field $\varphi(x^\mu, y)$ is decomposed into a tower of 4D fields $\varphi_n(x^\mu)$ as

$$\varphi(x^\mu, y) = \sum_{n=-\infty}^{\infty} \varphi_n(x^\mu) e^{iny/R}, \quad (3)$$

where the mass of the 4D field φ_n is given by $|n|/R$. The resulting spectrum is given in Fig. 1, where q correctively represents the standard-model quark and lepton fields. We find that there are many unwanted fields in the massless sector. In particular, the theory is necessarily vector-like reflecting underlying 5D Lorentz invariance (q is always accompanied by q^c), so that it cannot reproduce the standard model at low energies. Therefore, the simple compactification on S^1 clearly does not work.

What we have to do then is to compactify the extra dimension on an orbifold. The simplest orbifold called S^1/Z_2 is constructed by compactifying the extra dimension y on a circle of

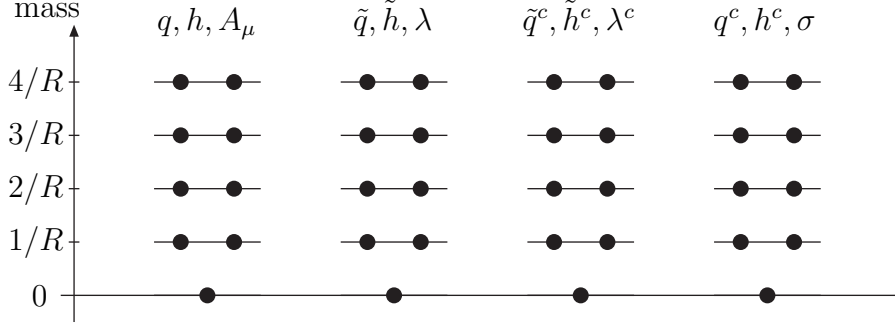


Figure 1: Tree-level KK mass spectrum of the matter, Higgs and gauge multiplets in S^1 compactification.

radius R and identifying points under the reflection $y \leftrightarrow -y$. Then, the physical space is an interval $y[0, \pi R]$ and has two special points at $y = 0$ and πR called fixed points. Under the reflection $y \leftrightarrow -y$, various 5D fields have definite transformation properties (parities), which are determined such that 5D Lagrangian is invariant under the parity transformation. We assign positive and negative parities for unconjugated and conjugated objects, respectively, in matter and Higgs multiplets. The transformations for the gauge multiplets are determined as $(+): A_\mu, \lambda$, $(-): \lambda^c, \sigma$.

The decomposition of 5D fields now goes through as follows:

$$(+) : \quad \varphi(x^\mu, y) = \sum_{n=0}^{\infty} \varphi_n(x^\mu) \cos \frac{ny}{R}, \quad (4)$$

$$(-) : \quad \varphi(x^\mu, y) = \sum_{n=1}^{\infty} \varphi_n(x^\mu) \sin \frac{ny}{R}. \quad (5)$$

The resulting mass spectrum is given in Fig. 2. We find that the half of the states are projected out by the identification $y \leftrightarrow -y$. Important point here is that, since $\sin 0$ is identically zero, the odd fields do not have zero modes. Thus, the structure of the massless sector is reduced to that of 4D $\mathcal{N} = 1$ SUSY and we obtain a chiral theory. However, it is not sufficient, since 4D $\mathcal{N} = 1$ SUSY is still remaining and we have massless superparticles. We have to do something more to obtain a completely realistic theory.

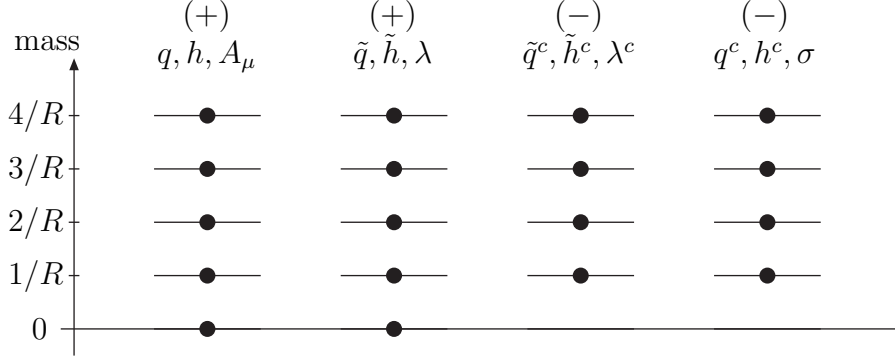


Figure 2: Tree-level KK mass spectrum of the matter, Higgs and gauge multiplets in S^1/Z_2 compactification.

3 Constrained Standard Model

We here propose to use the above orbifolding procedure twice to obtain a realistic theory [5]. Then, the second orbifolding projects out further half of the states, which breaks SUSY completely, and the matter content of the zero-mode sector is reduced to that of the standard model. Moreover, we can also introduce interactions which are precisely those of the one Higgs-doublet standard model, with the tree-level Higgs potential constrained as in Eq. (2) due to the underlying SUSY structure. Below, we will see how this works explicitly.

3.1 $S^1/(Z_2 \times Z'_2)$ orbifold

We consider a 5D $SU(3) \times SU(2) \times U(1)$ gauge theory and introduce hypermultiplets corresponding to three generations of matter, Q, U, D, L, E and a *single* Higgs field H . The extra dimension is compactified on the $S^1/(Z_2 \times Z'_2)$ orbifold, which is constructed by making two identifications, $Z_2 : y \leftrightarrow -y$ and $Z'_2 : y' \leftrightarrow -y' (y' = y - \pi R/2)$, on a circle of radius R . The physical space is now an interval $y[0, \pi R/2]$ which is a quarter of the circle. The 5D fields have definite transformation properties (parities) under the $Z_2 \times Z'_2$, and they are decomposed as

$$(+, +) : \quad \varphi(x^\mu, y) = \sum_{n=0}^{\infty} \varphi_{2n}(x^\mu) \cos \frac{2ny}{R}, \quad (6)$$

$$(+, -) : \quad \varphi(x^\mu, y) = \sum_{n=0}^{\infty} \varphi_{2n+1}(x^\mu) \cos \frac{(2n+1)y}{R}, \quad (7)$$

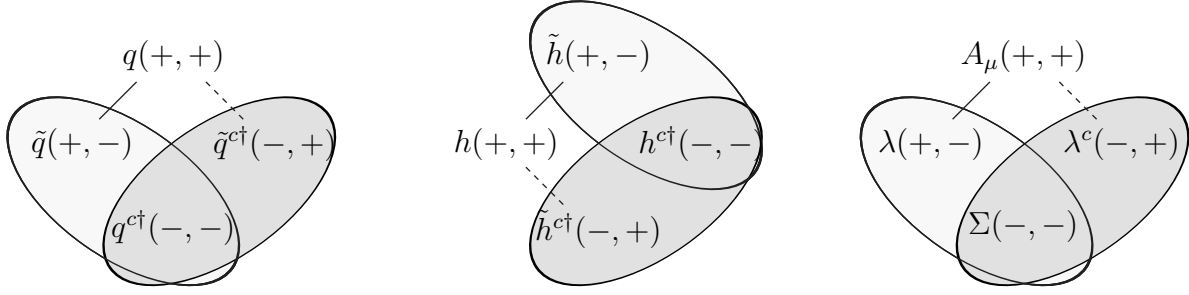


Figure 3: Quantum numbers of the matter, Higgs and gauge multiplets under the two orbifoldings $y \leftrightarrow -y$ and $y' \leftrightarrow -y'$. Here, $\Sigma = (\sigma + iA_5)/\sqrt{2}$.

$$(-, +) : \quad \varphi(x^\mu, y) = \sum_{n=0}^{\infty} \varphi_{2n+1}(x^\mu) \sin \frac{(2n+1)y}{R}, \quad (8)$$

$$(-, -) : \quad \varphi(x^\mu, y) = \sum_{n=0}^{\infty} \varphi_{2n+2}(x^\mu) \sin \frac{(2n+2)y}{R}, \quad (9)$$

according to their transformation properties under (Z_2, Z'_2) . The explicit parity assignment is given in Fig. 3. As we have seen in the previous section, the orbifolding by Z_2 projects out half of the states, which are inside the dark shaded ovals, from the zero-mode sector. However, now we have the other orbifolding by Z'_2 , which projects out different set of the states encircled by the light shaded ovals. As a result, if we make both orbifoldings simultaneously, the matter content of the zero-mode sector $((+, +)$ fields) reduces precisely to that of the standard model.

The KK spectrum of the model is given in Fig. 4. The zero-mode sector consists of the standard-model fields and, at the first excitation level, we have two superparticles for each standard-model particle reflecting underlying 5D SUSY structure. In particular, there are two superpartners (two Higgsinos) for the Higgs boson, which are massive forming a Dirac mass term, so that the theory is anomaly free. Note that we have evaded usual argument leading to two Higgs doublets simply because there is no energy interval where the 4D $\mathcal{N} = 1$ supersymmetric description is appropriate in this theory.

How about interactions? The Yukawa couplings can be introduced on the fixed points of the orbifold. We require that they preserve the symmetries which remain unbroken after the orbifold reflection about that fixed point. For instance, Z_2 orbifolding preserves 4D $\mathcal{N} = 1$ SUSY, so that the interactions placed on the $y = 0$ fixed point must preserve this symmetry. Choosing the hypercharge for h appropriately, we can place the supersymmetric Yukawa interactions $[QUH]_{\theta^2}$ on the $(3+1)$ -dimensional hypersurface at $y = 0$, which contain the up-type Yukawa couplings quh of the standard model. Here, Q , U and H represent chiral superfields under the

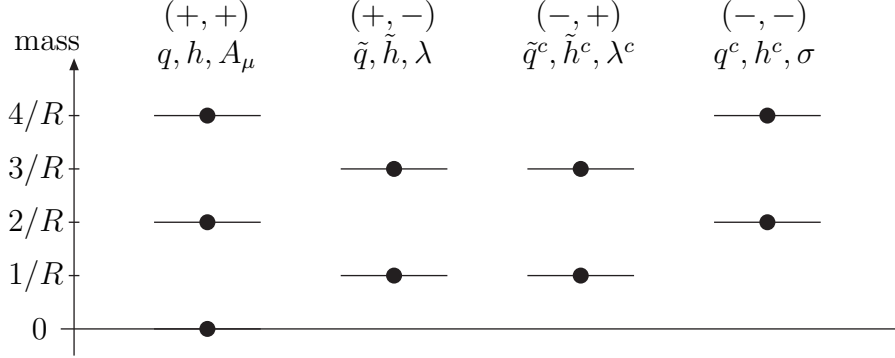


Figure 4: Tree-level KK mass spectrum of the matter, Higgs and gauge multiplets in $S^1/(Z_2 \times Z'_2)$ compactification.

4D $\mathcal{N} = 1$ SUSY that remains unbroken after the Z_2 orbifolding. Specifically,

$$Q = \tilde{q} + \theta q + \dots, \quad (10)$$

$$H = h + \theta \tilde{h} + \dots, \quad (11)$$

and they are represented by solid lines in Fig. 3. Similarly, we can also introduce interactions at the $y = \pi R/2$ fixed point. However, an important point here is that the remaining 4D $\mathcal{N} = 1$ SUSY after the Z'_2 orbifolding is different from that after the Z_2 . Thus, we must use *different* chiral superfields to write down the interactions at $y = \pi R/2$. They are denoted by dashed lines in Fig. 3, and given, for example, by

$$Q' = \tilde{q}^{c\dagger} + \theta' q + \dots, \quad (12)$$

$$H^{c'} = h^\dagger + \theta' \tilde{h}^c + \dots. \quad (13)$$

Then, the interactions we can write down at $y = \pi R/2$ are $[Q'D'H^{c'}]_{\theta'^2}$, which contain the down-type Yukawa couplings qdh^\dagger (and similarly for the charged leptons). Therefore, the interactions of the model are also those of the standard model, but the tree-level Higgs potential takes special form given in Eq. (2) since it only comes from the D -term potential of the gauge interactions.

3.2 Finite radiative electroweak symmetry breaking

Having established the tree-level structure of the model, we are now at the position of discussing radiative corrections. As usual, the dominant radiative correction to the Higgs potential comes

from the loops of the top quarks and squarks through the Yukawa couplings. Here, however, all the KK towers are circulating the loops, and it makes the radiative correction finite. The one-loop effective potential $V_{h,1\text{-loop}}$ is given by [5]

$$V_{h,1\text{-loop}} = \frac{18}{\pi^6 R^4} \sum_{k=0}^{\infty} \frac{\cos[2(2k+1) \arctan(\pi y_t R h/2)]}{(2k+1)^5}, \quad (14)$$

where y_t is related to the top-quark mass by $m_t = (2/\pi R) \arctan(\pi y_t R \langle h \rangle / 2)$. Note that the momentum integral leading to the above expression is completely dominated by $1/R$ scale, indicating that the physics of EWSB lies at the compactification scale. Furthermore, the summation of KK towers can also be cut off at a first few levels without affecting the result much. An important point here is that this cut-off procedure (regularization) must preserve correct symmetries of the theory [5]. Simply cutting off the summation at a finite level does not work, since it does not respect SUSY [9]. Appropriate regularizations have recently been discussed in Refs. [10].

The above points are elegantly understood by calculating the Higgs mass in the 5D mixed position-momentum space [8]. In this calculation, we work in the momentum space for usual 4D and the position space for the fifth dimension. We find that the diagrams in which the internal loops can continuously shrink to a point are cancelled between the bosonic and fermionic loops, and the only diagrams in which the internal propagators non-trivially wind round the extra dimension contribute to the Higgs mass. Then, since propagators are given by $\sim \exp(-p|y|)$, where p is the 4-momentum, the contributions from the modes with $p \gtrsim (\pi R)^{-1}$ are exponentially suppressed. (This precisely corresponds to the point-splitting regularization of a distance πR .) This shows that EWSB is caused by the physics at the compactification scale (boundary conditions) and exponentially insensitive to the physics at ultraviolet scales. An intuitive understanding of the result is the following. Since SUSY is broken by the boundary conditions, the Higgs boson could know this non-local breaking only when the internal top/stop propagators go around the circle. Then, it is clear that the radiative correction to the Higgs mass is finite, since the internal loops cannot shrink to a point where ultraviolet divergences would arise.

There is another way of understanding the finiteness of the Higgs mass. Remember that there are unbroken 4D $\mathcal{N} = 1$ SUSYs both at the $y = 0$ and $y = \pi R/2$ fixed points, although they are different subgroups of the original 5D SUSY. Likewise, we can also show that there are some residual SUSYs in *any* point in the extra dimension. (Of course, these SUSYs are different at different points in the extra dimension, so that there is no *global* SUSY after integrating out the extra dimension.) This means that we cannot write down a local operator which gives the Higgs mass. Therefore, if we calculate the Higgs boson mass, it must be finite since there is no counterterm for it. Here, by finite, we mean that there is no *intrinsic* divergence in the Higgs mass. Suppose that we calculate higher loop diagrams. Then, there are, of course, sub-divergences which give divergent contributions to the Higgs mass. However, they are all subtracted by supersymmetric counterterms. In other words, after renormalizing

all supersymmetric quantities such as gauge and Yukawa couplings, the expression of the Higgs mass is completely finite at all orders.

One final comment on the finiteness of radiative corrections is in order. From the above arguments, it should be clear that the finiteness of the Higgs mass relies on the fact that local SUSYs prevent to write the counterterm for the Higgs mass. This means that the quantities associated with SUSY breaking, *i.e.* the quantities which are vanishing in the SUSY limit, are all calculable in the present model. On the other hand, the quantities which can take nonvanishing values in the SUSY limit, such as the ρ parameter, are generically divergent and uncalculable [5].

3.3 Phenomenology

We can now calculate the compactification radius R and the physical Higgs boson mass $m_{\text{phys},h}$ by minimizing the Higgs potential obtained from Eq. (2) and Eq. (14). They are given by

$$\frac{1}{R} = (1+z)(352 \pm 20) \text{ GeV}, \quad (15)$$

$$m_{\text{phys},h} = 127 \pm 8 \text{ GeV}, \quad (16)$$

where z ($|z| \lesssim 0.2$) parameterizes unknown effects from ultraviolet physics and the errors come from 2-loop contributions etc [5]. This value of R is phenomenologically acceptable because there is no strong constraint coming from the direct production of KK gauge bosons or generation of four fermion operators in the models where the quarks and leptons propagate in the bulk [5, 11].

The masses for the superparticles are shifted from $1/R$ after EWSB. The largest effect appears on the top squark sector. Among the four top squarks (note that the number of superparticles are doubled compared with MSSM), two become heavier and two lighter after EWSB. The latter provides the lightest superparticle of mass

$$m_{\tilde{t}_-} = 197 \pm 20 \text{ GeV}. \quad (17)$$

Since these particles are stable, they produce characteristic signals in hadron colliders such as highly ionizing tracks and intermittent highly ionizing tracks [5]. Incidentally, the present limit on such particles is about 150 GeV from CDF [12].

4 Conclusions

If there is a TeV-sized extra dimension, we can construct supersymmetric theories with (i) no hidden sector (ii) finite radiative Higgs mass (iii) one Higgs doublet. In a minimal such model based on the $S^1/(Z_2 \times Z'_2)$ orbifold, the compactification radius and the physical Higgs boson

mass are determined as $1/R \simeq 350$ GeV and $m_{\text{phys},h} \simeq 127$ GeV. The lightest superparticle is a top squark of mass $m_{\tilde{t}_-} \simeq 197$ GeV, which may be seen at future linear collider experiments.

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